Name of Scholar: **Abdullah**

Name of Supervisor: **Prof. Iqbal Ahmed**

Name of Co- Supervisor: **Dr. K.B.Bhatnagar**

Department: **Mathematics**

Title of Thesis: **Periodic orbits of collision in the plane circular problem of four bodies**

Abstract

The entire work of this thesis has been divided in four chapters

The first chapter is introductory in nature. It contains history and development of the problem. The available literature is also reviewed from Euler (1760) to present day. In order to make the thesis self sufficient, we have defined some technical terms.

In the second chapter, we have studied the periodic orbits of collision in the plane circular problem of four bodies in which three primaries are placed on the vertices of an isosceles triangle and one primary is an oblate spheroid. This chapter is divided into ten sections. In the first section, we have given the brief description of the problem. In the second section, we have written the equations of motion in the cartesian form. In the third section, we have written the equations of motion in the complex form. In the fourth section, we have written Hamiltonian equations of motion for the regularized restricted problem. In the fifth section, we have written generating solutions of the equations of motion. In the sixth section, we have shown the existence of periodic orbits when $\mu \neq 0$, and also in the seventh section we have shown the existence of symmetric or doubly symmetric periodic orbits. In the eight section we have shown the collision of the periodic orbits by using Levi-civita (1904) condition.

In the ninth section, we have actually determined the periodic orbits and shown the effects of oblateness on the periodic orbits by numerical values given in the tables. In the tenth section, we have given the conclusion of the problem.

In the third chapter, we have studied the periodic orbits of collision in the plane circular problem of four bodies in which three primaries are placed on the vertices of a triangle and two of the primaries are oblate spheroid and all the primaries are source of radiation. This chapter has divided into ten sections, and has followed the procedure as done in chapter second.

In the fourth chapter, we have studied the periodic orbits of an artificial satellite moving under the gravitational forces of the sun including it`s solar radiation pressure, the moon and the oblate earth including it`s equatorial ellipticity. This chapter has been studied in seven sections. In the first section, we have given the brief description of the problem. In the second section, we have written the configuration of the problem. In the third section, we have given the different co-ordinate systems used in our problem. In the fourth section, we have written the angular motions. In the fifth section, we have determined the equations of motion in detail. These equations of motion are

$$\dot{\alpha} = \dot{\alpha}_0 + \sum_{i=1}^{182} A_i \text{Sin}\ \omega_i t,$$

$$\dot{\psi} = \dot{\psi}_0 + \frac{1}{\text{Sin}\alpha_0} \sum_{i=1}^{182} A_i \text{Sin}\ \omega_i t,$$

$$\ddot{r} - r\dot{\theta}^2 + \frac{GM_E}{r^2} = \xi + \sum_{i=1}^{7} (u_i + u_i' \cos 2\Gamma_0) \cos \omega_i t + \sum_{i=8}^{179} C_i \cos \omega_i t,$$

$$\frac{d}{dt}(r^2\dot{\theta}) = \eta + \sum_{i=1}^{7} (v_i + v_i' \cos 2\Gamma_0) \sin \omega_i t + \sum_{i=8}^{179} D_i \sin \omega_i t.$$

where,

$$\xi = -\frac{3GM_EJ_2R_0^2}{8r_0^4}\{2\text{-}3\sin^2(\varepsilon\text{-}\alpha_1)\}(2\text{-}3\sin^2\alpha_0)$$

$$+\frac{r_0\dot{\varphi}^2}{8}(2\text{-}3\sin^2\alpha_0)(2\text{-}3\sin^2\alpha_1)+\frac{\beta\, r_0}{R^3}$$

$$+\frac{r_0\dot{\theta}_M^2}{16\mu}(2\text{-}3\sin^2\alpha_0)(2\text{-}3\sin^2\alpha_1)(2\text{-}3\sin^2\alpha_m)$$

$$+\frac{9GM_EJ_2^2R_0^2}{4r_0^4}[(1+\cos^2\alpha_0)\{1+\cos^2(\varepsilon\text{-}\alpha_1)\}+2\sin^2(\varepsilon\text{-}\alpha_1)\sin^2\alpha_0]\cos(2\Gamma_0),$$

$$\eta = 6\dot{\theta}_0^2J_2^2R_0^2\cos(\varepsilon\text{-}\alpha_1)\cos(\alpha_0)\sin(2\Gamma_0).$$

Γ_0 = Steady state value of Γ,

ε = Obliquity,

α_m = Inclination of the moon's orbital plane to the ecliptic,

where α and ψ determine the orbital plane and r, θ determine the position of the satellite in the orbital plane. All the amplitudes u_i, v_i, C_i, D_i and frequencies ω_i are mentioned in the appendix. Since we are studying the in-plane motion, therefore, we will consider only the last two equations in (r, θ). We can supply the values of A_i, B_i on request. In the sixth section, we have discussed in detail about the periodic orbits and have actually drawn them. We have further shown the effects of earth`s equatorial ellipticity and solar radiation pressure on the satellite numerically. These effects are given in Tables [1(a, b, c, d), 2(a, b, c, d), c(a, b, c, d)], and finally we have concluded in the seventh section.