Abstract of Ph.D. Thesis

Title of Ph.D. Thesis: Investigation of Unexplored Aspects of Generalized Gaussian Hypergeometric Functions and Their Applications

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The present thesis comprises of six chapters and then each chapter is divided into a number of sections. Equations in every section are numbered separately. For example, in the small parentheses (a.b.c) the last figure denotes the equation number, the middle one the section and the first indicates the chapter to which it belongs. Sections, Articles, Definitions and Equations have been numbered chapter wise.

The aim of the chapter 1, is to introduce several classes of special functions, which occur rather frequently in the subsequent chapters. In this chapter, different forms of gamma function has been discussed. Legendre duplication and triplication formulae; Psi function; Polygamma function $\psi^{(m)}(x)$; Complete and Incomplete Beta and gamma functions; Fractional derivative; Hankel's contour integral formula; a useful limit formula for certain infinite products; Pochhammer symbol; Ordinary hypergeometric function of one variable ${}_AF_B$, its convergence conditions, properties associated with well-poised series, very well-poised series, Saalschützian series, nearly poised series of first and second kinds; Wright's generalized hypergeometric function ${}_p\Psi_q,\ {}_p\Psi_q^{\star}$; Hypergeometric summation theorems and Reduction formulae; Catalan's constant G; Kampé de Fériet's double hypergeometric function; Multiple ordinary hypergeometric function of Srivastava−Daoust and Some series identities, etc.

It provides a systematic introduction to most of the important special functions that commonly arise in practice and describes many of their salient properties. This chapter is intended to make the thesis as self contained as possible.

In chapter 2, some hypergeometric transformations for hypergeometric polynomial ${}_2F_1$, using integral operational techniques in multiplication formulas of Kummer's confluent hypergeometric function are given. Some hypergeometric summation theorems of Samtani and Bhatt are corrected by means of Gauss second summation theorem. Using series rearrangement techniques, an expansion formula has been obtained.

In chapter 3, by making use of the result of Slater [92] associated with truncated hypergeometric series, which have been given in the correct form by Agarwal [3, p.18(11)] (there were some errors in the conditions of the main result given by Slater) we compute, some interesting summation of certain series both, truncated as well as infinite. The summations for truncated hypergeometric series $_4F_3[1]_N$, $_5F_4[1]_N$, $_6F_5[1]_N$ and $_7F_6[1]_N$ and their infinite versions $_4F_3[1]$, $_5F_4[1]$, $_6F_5[1]$ and $_7F_6[1]$ with real and complex values of numerator and denominator parameters has been found.

Chapter 4 begins with the classical hypergeometric proof of an identity by Lyons, Paule and Riese [65]. The identity was verified by means of computer algorithm and determinantal process; that arose in the work of Lyons and Steif [66] on determinantal processes.

For a curious result of Srinivasa Ramanujan, a proof has been given by K. Ramachandra [85], in order to derive the result on continued fractions. An alternative proof involving hypergeometric functions for the same result has been presented.

Chapter 5 and 6 are associated with the applications of hypergeometric functions in solving some interesting problems of physics.

In classical mechanics, the problem of a simple harmonic oscillator, modified by the presence of a non harmonic term, is customarily solved by perturbative methods. These methods yield approximate solutions which may not be represented in a closed form. In the present study, the problem to be investigated concerns the classical motion of a point particle under the action of the central potential ar^N, where index N is arbitrary. It is shown that exact solutions of the problem may be obtained by the application of hypergeometric functions. The method is used for potentials more general than the harmonic oscillator one. The results agree, in the lowest order, with those obtained (by H. Goldstein for instance) by the usual approximate methods.

The motion of a simple pendulum of arbitrary amplitude is usually treated by approximate methods. By using generalized hypergeometric functions, it is however possible to solve the problem exactly. In the present work, the exact equation of motion of a simple pendulum of arbitrary amplitude has been provided. The time period of such a pendulum is also exactly expressible in terms of hypergeometric functions. The exact expressions thus obtained, are used to calculate numerical values of the time period; for arbitrary amplitudes.

A detailed bibliography appears at the end; with the author names in alphabetical order. References to the bibliography are numbered. The thesis also includes appendices which contains reprints of few published papers and Gamma tables, etc (see the hardcopy version).