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Title of the Thesis: Generating Functions and Expansions of Generalized Special Functions

The purpose of the present research was to study generalized functions, which can provide a unification scheme for the well known special functions (scattered in the existing literature). Especially the generalization of the special functions which are widely used in various areas of applied mathematics have been dealt with. The results thus established do not merely generalize the results given earlier by various workers but also yield a number of new results. The findings can be applied to physics, statistics, engineering and other branches of science.

The present thesis comprises of EIGHT CHAPTERS. A brief summary of the problems is presented at the beginning of each chapter and then each chapter is divided into a number of sections.

The aim of the CHAPTER 1, is to introduce several classes of special functions, which occur rather frequently in the subsequent chapters. In this chapter, different forms of gamma function, Legendre duplication and triplication formulae; Psi function; Polygamma function $\psi^{(m)}(x)$; Complete and Incomplete Beta and gamma functions; Fractional derivative; Hankel's contour integral formula; A useful limit formula for certain infinite products; Pochhammer symbol; Ordinary hypergeometric function of one variable ${}_AF_B$, its convergence conditions, properties associated with well-poised series, very well-poised series, Saalschützian series, nearly poised series of first and second kinds; Wright's generalized hypergeometric function ${}_p\Psi_q$, ${}_p\Psi_q^\star$; Hypergeometric summation theorems and Reduction formulae; Catalan's constant G; Kampé de Fériet's double hypergeometric function; Multiple ordinary hypergeometric function of Srivastava−Daoust and Some series identities, etc. have been discussed.

It provides a systematic introduction to most of the important special functions that commonly arise in practice and describes many of their salient properties. This chapter is intended to make the thesis as self contained as possible.

CHAPTER 2, concerns mainly with verification and extension of the table for $\tau(1)$, $\tau(2)$, $\tau(3)$, $\ldots$, $\tau(30)$ of Ramanujan. Our extended table for $\tau(31)$, $\tau(32)$, $\tau(33)$, $\ldots$, $\tau(211)$ is obtained without using certain arithmetical functions defined by Ramanujan and also the theory of elliptic functions.

The main object of CHAPTER 3 is to evaluate three definite integrals of Gradshteyn-Ryzhik with certain convergence conditions, using Leibnitz rule for differentiation under integral sign and Wallis formula. Some other integrals are also evaluated by means of Leibnitz rule, Kummer's first transformation and reduction formula, series rearrangement techniques under stated convergence conditions.

In CHAPTER 4, we obtain a mixed theorem on Laplace and inverse Laplace transforms. We also find some consequences of mixed theorem in terms of Fox-Wright hypergeometric

function and generalized Gaussian hypergeometric function.

Furhter we obtain a general theorem on Laplace transform in terms of Fox-Wright hypergeometric function. Laplace transforms of some composite functions are also obtained as special cases.

The main object of CHAPTER 5 is to develop some applications, extensions and generalizations of Ramanujan's integral (which he obtained with the help of the celebrated Ramanujan Master theorem). Motivated essentially by some recent works by M. Garg and S. Mittal and H. M. Srivastava et al., we evaluate several general families of new definite integrals involving functions of one and more complex variables, some of which are then applied to derive the corresponding results associated with generalized Gaussian hypergeometric function ${}_pF_q$ and the Fox-Wright generalized hypergeometric functions ${}_p\Psi_q,\ {}_p\Psi_q^*$.

In CHAPTER 6, we shall discuss Maclaurin's expansions and Mellin transforms of some composite functions, using Ramanujan's Master theorem.

In CHAPTER 7, we obtain successive differentiation and change of argument of Gould-Hopper polynomials. We generalized Curzon's integral for Legendre's polynomial of first kind. Some multivariable linear, bilinear and bilateral generating relations are also obtained, using series iteration technique.

CHAPTER 8, mainly concerns with three theorems involving generating functions expressed in terms of single and double Laplace and Beta integrals. These theorems have been applied to obtain bilinear and bilateral generating functions involving polynomials of Mittag-Leffler, Madhekar-Thakare, Gottlieb, Jacobi, Konhauser, Laguerre and other polynomials hypergeometric in nature. One variable special cases of the hypergeometric polynomials are important in several applied problems.