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Abstract

Special functions and their applications are now awe-inspiring in their scope, variety and depth. Not only in their rapid growth in pure Mathematics and their applications to the traditional fields of Physics, Engineering and Statistics but in new fields of applications like Behavioral Science, Optimization, Biology, Environmental Science and Economics, etc. they are emerging.
A special function is a real or complex valued function of one or more real or complex variables which is specified so completely that its numerical values could in principle be tabulated. Besides elementary functions such as x^n, e^x, $\ell_n(x)$, and $\sin x$,“higher” functions, both transcendental (such as Bessel functions) and algebraic (such as various polynomials) come under the category of special functions. In fact, special functions are solutions of a wide class of mathematically and physically relevant functional equations. As far as the origin of special functions is concerned the special function of mathematical physics arises in the solution of partial differential equations governing the behavior of certain physical quantities.
Multiple hypergeometric functions constitute a natural generalization of the Gauss hypergeometric functions of one variable. Since the introduction of double hypergeometric functions by Appell and triple hypergeometric functions by Lauricella, numerous papers by many workers have been published and the theory has been considerably extended. An extensive study has been made in Europe, America and India for these multiple functions, which produced an explosion of knowledge of the subject.
In most sciences one generation tears down what another has built, and what one has established another undoes. In **Mathematics** alone each generation builds a new storey to the old structure.
The present thesis comprises of **Eight Chapters**. A brief summary of the problems is presented at the beginning of each chapter and then each chapter is divided into a number of sections. Equations in every section are numbered separately. For example the small bracket (a.b.c) specified the results, in which the last figure denotes the equation number, the middle one the section and the first indicate the chapter to which it belongs. Section, Articles, Definitions and Equations have been numbered chapter wise.
The aim of the **First Chapter** is to introduce several classes of special functions, which occur rather more frequently in the study of summations and transformations needed for the presentation of subsequent chapters. In this chapter, we have discussed different forms of Gamma function; Legendre duplication and triplication formulae; ordinary hypergeometric function of one variable and its convergence conditions; ordinary double hypergeometric function of Exton and Kampé de Fériet's; power series form, convergence conditions and associated results; Appell's function.
It provides a systematic introduction to most of the important special functions that commonly arise in practice and explores many of their silent properties. This chapter is also intended to make the thesis as much self contained as possible.

In **Chapter 2**, we derive summation theorems for the Appell's function $F_1[A;\ B,\ C;\ D;\ 1,\ y]$, $F_1[A;\ B,\ C;\ D;\ x,\ x]$, $F_1[A;\ B,\ B;\ D;\ x,\ -x]$ and $F_1[A;\ B,\ B;\ 2B;\ x,\ -x]$ by using suitable reduction formulas.

In **Chapter 3**, we derive summation theorems for the Appell's function $F_2[A;\ B,\ -m;\ D,\ G,\ 1,\ y]$ and $F_2[-m;\ B,\ C;\ D,\ G,\ 1,\ y]$ with suitable convergence conditions.

In **Chapter 4**, we derive the summation theorems involving Appell's hypergeometric functions F_3. Applications in summation theorems for $F_3[A,\ B;\ C,\ D;\ G;\ 1,\ y]$, $F_3[A,\ A;\ B,\ B;\ A+B;\ x,\ -x]$ and $F_3[A,\ B;\ 1,\ 1;\ A+B;\ x,\ y]$ are also established.

In **Chapter 5**, we obtain the summation theorems involving Appell's hypergeometric functions $F_4[-m,\ B;\ B,\ D;\ 4,\ 1]$, $F_4[-2m,\ B;\ B,\ 1-2m-B;\ x,\ -1]$, $F_4[-1-2m,\ B;\ B,\ -2m-B;\ x,\ -1]$, $F_4[A,\ B;\ C,\ D;\ x,\ y]$, $F_4[A,\ B;\ C,\ 1+A+B-C;\ x(1-y),\ y(1-x)]$ and $F_4[A,\ B;\ C,\ C;\ x,\ -x]$.

In **Chapter 6**, we obtain certain reduction formulas involving Appell's hypergeometric functions $F_1[A\,;\ B,\ C\,;\ B{+}C\,;\ x,y]$, $F_2[A\,;\ B,\ C\,;\ A,\ A;x,y]$ and $F_2[A\,;\ B,\ C\,;\ B\ ,\ D;\ x,y]$.

In **Chapter 7**, certain reduction formulas related to Appell's functions $F_3[A\ ,\ C-A;\ B\ ,\ C-B;\ C;x,\frac{-y}{1-y}]$, $F_4[A\ ,\ B;\ B\ ,B\ ;\frac{-x}{(1-x)(1-y)},\frac{-y}{(1-x)(1-y)}]$ and $F_4[A\ ,\ B;\ 1+A-B\ ,\ B\ ;\ \frac{-x}{(1-x)(1-y)},\frac{-y}{(1-x)(1-y)}]$, are derived.

Chapter 8 is related with conclusion.