# ABSTRACT

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**Title of the thesis:** Some Contribution to Rings & Near Rings
**Keywords:**Commutators, Derivation, Generalized Derivation, Γ-Derivation, Generalized Γ-Derivation, Ideals, $\sigma$-Ideals, Ring, Prime Ring, Semiprime Ring, $\sigma$-Prime Ring, Nearring, Prime Nearring, Γ-Nearring, Prime Γ-Nearring.

The growing importance of the derivations in certain classes of ring and nearring, is the motivation behind selecting the topic for the present thesis. Most of the chapters of this thesis have either published or accepted for publication. The present thesis comprises five chapters. The first chapter is introductory, where we have given some definitions and examples from the theory of ring and nearring which shall be use subsequently in the thesis.

In **Chapter II**, we have studied the relationship between the commutativity of a ring $R$ and the behavior of a special mapping which are centralizing or commuting on some appropriate subsets of $R$. The history of commuting and centralizing mappings goes back to 1955 when Divinsky[Trans. Roy. Soc. Canada. Sect. III 49 (1955), 19-22] proved that a simple artinien ring is commutative if it has a commuting non trivial automorphism. Later, Posner [Proc. Amer. Math. Soc. 8 (1957), 1093-1100] has proved that the existence of a nonzero centralizing derivation on a prime ring forces the ring to be commutative (Posner Second Theorem). It is a fact that this result has been extremely influential and it has been generalized by a number's of authors in several ways. Motivated by the above work, we have developed the study of subject in this direction by providing some results which are of independent interest and are related to derivations on some appropriate subsets of a ring. At the end of the chapter, we have given some examples to show that the hypothesis of several results of this chapter are not true for the case of a arbitrary ring. The contents of this chapter are published and accepted in **Pure Mathematics, IASR, 4(2012), 88-95 and Kyungpook Mathematical Journal Volume 53 (4) (2013), 565-571** respectively.

**Chapter III** deals with the generalized derivations on some appropriate subsets of a $\sigma$-prime ring. The concept of $\sigma$-prime ring has been introduced by Oukhtite [Afr. Diaspora J. Math. 5 (2006), 19-23]. However, the actual motivation behind their first successfully work came from Posner[Proc. Amer. Math. Soc. 8 (1957), 1093-1100] second theorem only. In 2006 [Int. J. Contemp 1 (2006), 439-448], they successfully extended the result for $\sigma$-prime ring. Recently, a major breakthrough has been achieved by Oukhtite et al.[Int. J. Algebra 1 (2007), 241-246], where the important results by Posner, Herstein and Bell have been proved for $\sigma$-prime rings. After this breakthrough, they extended several well known results valid for a prime ring by taking a $\sigma$-prime ring into consideration. A continuous approach in this direction is still on. In this chapter, we have

developed the study of the subject in the direction of Oukhtite by providing some results which are of independent interest and are related to generalized derivations on $\sigma$-ideals of a $\sigma$-prime ring. Further some examples are given to show that the hypothesis of several results of this chapter are not true for the case of a arbitrary ring. The results of this chapter are accepted for publication in **ISRN Algebra, Volume 2013, Article ID 572690, 5 Pages** .

In **Chapter IV**, we have studied the relationship between the commutativity of a nearring and the behavior of a special mapping on nearring. The literature on nearrings contains a number of theorems asserting that the certain conditions implying commutativity in rings imply multiplicative or additive commutativity in special classes of nearrings. It is well known fact that prime, semi prime nearrings with derivations have ring like behavior and this has been done by a number's algebraists. In 1992, Bell and Mason [Math. J. Okayama Univ. 32 (1992), 135-144] proved that the results of [Math. J. Okayama Univ. 32 (1992), 135-144] obtained by replacing a derivation with a generalized derivation is still true. In [Journal of Advanced Research in Pure Mathematics 3 (2011), 120-124], the authors have obtained the same results for a generalized derivation. Inspired by the same, we have extended the results of [Journal of Advanced Research in Pure Mathematics 3 (2011), 120-124] for a noncommutative prime nearring and established some results which are of independent interest and are related to generalized derivations on some appropriate subsets of a nearring. At the end of the chapter, we have given some examples to show that the hypothesis of several results of this chapter are not true for the case of a arbitrary ring. The contents of this chapter are accepted for publication in **European Journal of Mathematical Sciences** and **Chinese Journal of Mathematics, Volume 2013, Article ID 752928, 4 pages** .

The study of derivations in $\Gamma$-nearrings has been introduced by Jun, Cho and Kim [Indian J. Pure and Appl. Math. 33 (2002) 1489-1494 & Soochow J. Math. 29 (2003), 275-282] and have investigated the several basic properties of $\Gamma$-derivations in their papers. Thereafter, UcCkun et. al. [Commun. Korean Math. Soc. 19 (2004), 427-433] worked on prime $\Gamma$-nearrings with $\Gamma$-derivations and they found conditions for a $\Gamma$-nearring to be commutative. As there were only few papers on $\Gamma$-derivations of $\Gamma$-nearrings and none (to knowledge of author) on generalized $\Gamma$-derivations of $\Gamma$-nearrings. The concept of generalized $\Gamma$-derivation in $\Gamma$-nearring is given in **Chapter V** and to initiate the study of the theory, the existence of such $\Gamma$-derivation is shown by an example. Further, we have extended the results of [Soochow J. Math. 29 (2003), 275-282 & Commun. Korean Math. Soc. 19 (2004), 427-433] by replacing a $\Gamma$-derivation with a generalized $\Gamma$-derivation and given some examples to show that the hypothesis of several results of this chapter are not true for the case of a arbitrary ring. The contents of this chapter are accepted for publication in **Novi Sad Journal of Mathematics**.