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### ABSTRACT

Keywords : Immersion, Submersion, Spacelike submanifold, Locally conformal Kaehler manifold, Generalized complex space form.

The present thesis comprises six chapters and each chapter is divided into various sections. The **first chapter** is introductory in nature where we have collected definitions and results which are of relevance to the subsequent chapters.

The **second chapter** of the thesis is devoted to the study of geometry of pseudo-umbilical subspaces immersed in a space form. Second section deals with first nonzero eigen value of the Laplacian operator of a pseudo-umbilical hypersurface immersing in a unit sphere  $S^{n+1}$  generalizing results due to Deshmukh [7] proved in case of minimal hypersurface. The third section deals with pseudo-umbilical spacelike submanifold  $M^n$  of an indefinite space form  $M_p^{n+p}(c)$ . *This chapter is partially accepted in Russian Mathematics (Springer).*

The **third chapter** is devoted to investigate submersion of CR-submanifolds of a l.c.q.K manifold  $\widetilde{M}$ . Using a result due to simons [4] we prove that if an almost quaternion manifold  $B$  admits a Riemannian submersion  $\pi : M \rightarrow B$  of a CR-submanifold  $M$  of a l.c.q.K manifold  $\widetilde{M}$ , then  $B$  is a l.c.q.K manifold. Relations between curvature tensors of  $M$  and  $B$  and holomorphic sectional curvatures of  $B$  and  $\widetilde{M}$  are also obtained. *The content of this chapter is already published in ISRN geometry (2012).* In **fourth chapter**, we extend the notion of anti-invariant and Langrangian Riemannian submersion (introduced by B. Sahin [2]) to the case when the total manifolds are l.c.K. and l.c.q.K. manifolds. We obtain some integrability conditions for the horizontal distribution and discuss necessary and sufficient condition for a Langrangian Riemannian submersion to be totally geodesic.

Totally real submanifolds have been studied in different ambient spaces by many geometers

e.g. B. Y. Chen, K. Ogiue [3], K. Yano, M. Kon [5]. In **fifth chapter**, we have extended this notion and studied totally real submanifolds in generalized complex space form. We derive some integral formulas computing the Laplacian of the square of the second fundamental form and using these formulas we prove a pinching theorem. Warped product manifolds have applications in physics. For instance, they are widely used to provide settings to model space time near black holes or bodies with large gravitational force. In view of their growing applications, we have devoted **sixth chapter** to study the problems of existence (or non-existence) of warped product manifolds in various ambient manifolds assumes significance in general and has been studied by Sahin [1]. So, of warped product submanifolds in l.c.K. and l.c.q.K. manifolds.

## References

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