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Topic of Research: "A Study of Fractional Ideals and Ideal Class Group in Algebraic System"

Findings

Fractional ideals are a crucial concept in commutative algebra, essentially ideals in rings to their field of fractions, allowing for elements with denominators but maintain structure. The main aim of the thesis is to investigate the structural and algebraic foundations of soft and fuzzy fractional ideals in integral domains, focusing on their interplay with fraction fields and their significance in shaping core algebraic structures. The study introduces new operations, such as soft star operations, soft semistar operations, and soft star ideals to establish a complete lattice structure which provide a comprehensive framework for characterizing integral domains including Dedekind domains and Noetherian domains. A key concept in the study of soft fractional ideals is the existence of finite minimal generating set (as a soft R-submodule). The study further explores the invertibility and factorization of fuzzy and soft fractional ideals, leading to unique decomposition results crucial for algebraic analysis. By applying fuzzy set and soft set methodologies, this research also develops innovative approaches to localization and discrete valuation rings, enriched with graphical representations that enhance both computational and theoretical understanding. The thesis consists of seven chapters; an outline of each chapter is provided as follows:

***Chapter* 1** serves as a foundation for the entire thesis, provides an overview of ideal theory along with a discussion of the properties of ideals in different algebraic structures. It also covers various procedures related to the fractional ideals in multiplicative ideal theory. Additionally, fundamental definitions and examples of several types of ideals and modules are provided. Basic concepts and preliminaries about fractional ideals, as well as a discussion of fuzziness and softness of ideals are included.

***Chapter* 2** is dedicated to the study of the valuation rings in relation to soft modules of the fraction field and also proved the equivalence of undeniable soft modules of the fraction field, called soft fractional ideals. Drawing attention to the properties of multiples of soft modules, soft submodules of the fraction field, and soft fractional ideals of an integral domain.

***Chapter* 3** is devoted to the notion of soft star-operations and soft star-ideals which are crucial to the investigation of soft fractional ideals of integral domains. Introducing the new concept and showing that how the complete lattice is formed by the set of all soft star operations in relation to soft fractional ideals on integral domain. By applying these concepts and freshly defined concepts, several integral domains including pseudo principal domains, principal ideal domains, and greatest common divisor domains are characterized.

***Chapter* 4** deals with the concept of soft semistar operations and their role in the study of soft fractional ideals of integral domains. Using the notion of quotient overring, some extensions of soft fractional ideals are demonstrated. Introducing the new concept, how the complete lattice is formed by the set of all soft semistar operations in relation to soft fractional ideals on integral domain and connect it to the current notions of star and semistar operations. Using these concepts and freshly defined concepts, some illustrations including soft d-semistar operation, soft e-semistar operation and soft v-semistar operation are provided.

***Chapter* 5** provides a foundation for innovative approaches to the study of soft multiplicative ideal theory. A key concept in the study of soft fractional ideals is the existence of finite minimal generating set (as a soft R-submodule). Introducing the novel concept and demonstrate that for a finite valued soft fractional ideal of integral domain with invertible level ideals, there exists a finite minimal generating set (as a soft R-submodule). Furthermore, we investigate the invertibility of specific soft fractional ideals and the factorization of soft ideals into products of prime and maximal soft ideals, through which the characterizations of Dedekind domains are established.

***Chapter* 6** focuses on the unique factorization of fuzzy fractional ideals in Dedekind domains. For the localization of a Noetherian domain at its fuzzy fractional ideals, some intermediate results are investigated. The study examines how the undeniable fuzzy fractional ideals can be factorized uniquely in Dedekind domains. Furthermore, we show that the localization of a Noetherian domain at its fuzzy fractional ideals is a discrete valuation ring, as demonstrated by the newly introduced ideas and several examples.

***Chapter* 7** focuses on the study of soft fractional ideals in Dedekind domains, heavily relies on the concept of unique factorization. Some intermediate results for the localization of a Noetherian domain at its soft fractional ideals are demonstrated. The study examines how the newly introduced notions and examples demonstrate that the localization of a Noetherian domain at its soft fractional ideals is a discrete valuation ring, supported by graphical representations.