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Topic: Some Contributions to Quasi Covered Ideals in Algebraic Structures and Hyperstructures

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Findings

The research work entitled “**Some Contributions to Quasi Covered Ideals in Algebraic Structures and Hyperstructures**” presents a systematic investigation of quasi covered ideals, quasi bases, hyperideals and covered bi-hyperideals in different algebraic structures and hyperstructures. The major findings of the research are summarized below.

1. Quasi Covered Ideals in Semigroups

A central contribution of the research is the introduction and systematic study of quasi covered ideals in semigroups as a generalization of covered ideals. Their fundamental algebraic properties and behavior under standard set operations, particularly intersection and union, have been investigated. The relationships among quasi covered ideals, covered ideals, maximal ideals, and greatest ideals have also been established.

The study introduces the concepts of greatest quasi covered ideals and quasi bases, which provide useful tools for analyzing the internal structure of semigroups. Conditions are established under which greatest ideals can be regarded as quasi covered ideals. The role of quasi bases in characterizing the structure of semigroups is examined through theoretical results and illustrative examples.

2. Quasi Covered Ideals in Ordered Semigroups

The research further extends the theory to ordered semigroups, where algebraic operations are accompanied by a compatible partial order. In this setting, quasi covered ideals and quasi bases are formulated by incorporating order theoretic properties into the algebraic framework.

Several characterizations of quasi covered ideals in ordered semigroups are obtained. The research establishes precise relationships between quasi covered ideals, covered ideals and greatest ideals. In particular, conditions are identified under which a covered ideal or a greatest ideal of an ordered semigroup becomes a quasi covered ideal.

The study also develops the notions of quasi bases and greatest quasi covered ideals of ordered semigroups and analyzes their mutual relationship. These results demonstrate that the underlying partial order has an important influence on the structure and behavior of quasi covered ideals.

3. Quasi Covered Hyperideals in Semihypergroups

Another major finding is the extension of quasi covered ideal theory from ordinary algebraic structures to semihypergroups. Since hyperoperations may produce a set of elements rather than a single element, semihypergroups provide a broader framework for studying generalized algebraic operations.

The research introduces quasi covered hyperideals as a natural extension of covered hyperideals in semihypergroups and establishes their fundamental properties. Their relationships with covered hyperideals and greatest hyperideals are investigated. The concepts of quasi hyperbases and greatest quasi covered hyperideals are also introduced and analyzed.

Furthermore, the research establishes characterizations involving principal bi-hyperideals and investigates the existence of minimal, maximal and greatest covered bi-hyperideals. Structural properties of covered bi-hyperideals in regular semihypergroups are also examined.

4. Quasi Covered Hyperideals in Ordered Semihypergroups

The theory is further generalized to ordered semihypergroups, combining the effects of hyperoperations and compatible partial orders. Quasi covered hyperideals and quasi hyperbases are defined and studied in this more general setting.

The research establishes fundamental properties of quasi covered hyperideals associated with order preserving hyperoperations. Relationships between covered hyperideals, greatest hyperideals, maximal hyperideals, and quasi covered hyperideals are clarified. Conditions are established under which greatest hyperideals become quasi covered hyperideals.

A significant finding is that the order structure plays an important role in determining the existence and uniqueness of greatest quasi covered hyperideals. The relationship between quasi hyperbases and greatest quasi covered hyperideals is also established, showing how the structure of a quasi hyperbase influences the formation and properties of the corresponding greatest quasi covered hyperideal.

5. Quasi Covered Ideals in Ternary Semigroups

The research also extends the concepts of quasi covered ideals and quasi bases to ternary semigroups, where the fundamental operation is an associative ternary operation rather than a binary operation.

Quasi covered ideals in ternary semigroups are introduced and their structural properties are investigated through theoretical results and examples. Particular attention is given to their behavior under intersection and union.

The study also examines the existence and non-existence of greatest quasi covered ideals and quasi bases in ternary semigroups. Conditions for the existence of greatest quasi covered ideals are developed, thereby extending the generalized ideal framework to another important class of algebraic systems.

6. Overall Contribution and Significance

Overall, the research develops a framework for studying quasi covered ideals and related generalized ideal structures across semigroups, ordered semigroups, semihypergroups, ordered semihypergroups and ternary semigroups. The work establishes new definitions, structural properties, characterizations, existence conditions and relationships among quasi covered ideals, quasi bases, greatest ideals, maximal ideals, hyperideals and covered bi-hyperideals.

In conclusion, the findings establish quasi covered ideals and quasi bases as useful generalized concepts for investigating the structure of algebraic systems. The results provide a foundation for further investigations involving generalized hyperoperations, fuzzy and intuitionistic fuzzy structures.

Keywords: Semigroup, Quasi Covered Ideal, Covered Ideal, Quasi Base, Greatest Ideal, Ordered Semigroup, Semihypergroup, Hyperideal, Covered Bi-Hyperideal, Ordered Semihypergroup, Quasi Covered Hyperideal, Quasi Hyperbase, Ternary Semigroup, Regular Semigroup.