Name of the Scholar: Pooja Bansal

Name of the Supervisor: Prof. Mahammad Hasan Shahid

Department: Department of Mathematics,
Faculty of Natural Sciences,
Jamia Millia Islamia,
New Delhi-110025, INDIA.

Title: A Study of Geometry of Submanifolds

Abstract

Submanifold theory plays an important role in the development of modern differential geometry. It is a very active and vast research field. Submanifold theory is still far from being exhausted; only a small portion of an exceedingly fruitful field has been cultivated and much more remains to be discovered. The present thesis has been written on the same basis and efforts were made to contribute something to the subject which are well motivated.

The thesis "A Study of Geometry of Submanifolds" is divided into eight chapters, minuscule outline of which are as follows:

Chapter 1. This chapter is fully introductory, here we describe basic definitions, formulae and results which are relevant to the subsequent chapters. Although most of these results are available in standard references on the subject, nevertheless we have collected them to shape the thesis self contained.

Chapter 2. This chapter deals with the study of relationship between intrinsic and extrinsic invariants of real hypersurfaces of complex quadric. First we establish B. Y. Chen's inequalities for real hypersurfaces of complex quadric and by considering the equality case, we obtain some consequences. Also, we establish an inequality in terms of the warping function and the scalar curvature for a warped product real hypersurface of complex quadric and some obstructions have been given. Next, we study real hypersurface of complex quadric admitting semi-symmetric metric connection and find the curvature tensor. Additionally, using this curvature we develop inequalities involving Chen δ-invariant admitting a semi-symmetric metric connection. Finally, the equality case is considered. *Results of this chapter have been published in* **International Electronic Journal of Geometry**, **11**(2) (2018), 34-46.

Chapter 3. In this chapter, we first presented general results of real hypersurfaces in complex quadric and characterize real hypersurfaces in complex quadric by giving an inequality concerning the scalar curvature and the mean curvature vector field. We also obtain the condition under which this inequality becomes an equality. Moreover in a continuation, we develop two extremal inequalities involving the normalized δ-Casorati curvatures and the extrinsic generalized normalized δ-Casorati curvatures for real hypersurfaces in complex quadric. Finally, we derive the necessary and sufficient condition for the equality in both cases. Next we develop similar results for real hypersurfaces in complex quadric admitting semi-symmetric metric connection together with the derivation of the

necessary and sufficient condition for equality case. *Results in this chapter have been published in* **Tamkang Journal of Mathematics**, **50**(2) (2019), 187-198 and **Arabian Journal of Mathematics**, (2018), 1-11.

Chapter 4. This chapter is devoted to obtain DDVV conjecture (generalized Wintgen inequality) for statistical submanifold of Kenmotsu statistical manifold of constant φ-sectional curvature. Firstly, we obtained some general results on Riemannian curvature and Ricci tensor. Next, we establish general geometric inequality for statistical submanifold of Kenmotsu statistical manifold of constant φ-sectional curvature. Finally, we develop DDVV conjecture (generalized Wintgen inequality) involving normalized scalar curvature and normalized scalar normal curvature with Casorati curvature for same space. Lastly, we provide glimpse of applications of derived inequality. *Results in this chapter have been published in* **Journal of Geometry and Physics**, **142** (2019), 37-46.

Chapter 5. The first section of this chapter construct some general geometric inequalities for statistical submanifold of Kenmotsu statistical manifold of constant φ-sectional curvature. Later, we obstruct Chen-Ricci inequality and B. Y. Chen inequality for submanifolds with any codimension in Kenmotsu statistical manifold and some consequences of derived inequalities are also described. In section two we mainly derive the B. Y. Chen inequality for statistical submanifolds of statistical warped product manifolds of the form $R \times_f N$ with glimpse of its geometric applications. Next section develop B. Y. Chen inequality for Legendrian statistical submanifold in statistical warped product manifolds $R \times_f N$. We also provide some applications of derived inequalities in a statistical warped product manifold which is equivalent to a hyperbolic space. In last section, we construct new examples in statistical warped product manifolds. *Results of this chapter have been communicated for publication* **collaborated with Bang Yen Chen**.

Chapter 6. In this chapter, we study statistical submanifolds of statistical manifold of quasi-constant curvature initiating with general results on scalar curvature. In next section, we acquire optimal extremities for statistical submanifold of statistical manifold of quasi-constant curvature associating the normalized scalar curvature and the extrinsic generalized normalized δ-Casorati curvatures. Additionally, the equality cases are also considered. Last section extract Chen first inequality for statistical submanifold of statistical manifold of quasi-constant curvature and also give some glimpse of applications of derived results. *Results in this chapter have been communicated for publication*.

Chapter 7. In this chapter, we first study locally conformal almost Cosymplectic manifold admitting semi-symmetric metric connection. Next, we deduce extremities for the generalized normalized δ-Casorati curvatures of submanifolds in same ambient manifold admitting semi-symmetric metric connection using T. Oprea's technique. Eventually, applying on extremities, applications of these inequalities give rise to several inequalities for different type of submanifolds (like bi-slant, hemi-slant, semi-slant, slant, invariant, anti-invariant, CR-submanifolds). Moreover, we characterizes submanifolds on which equalities hold. Finally, we establish an inequality in terms of the warping function and the Casorati curvature for warped product submanifold of locally conformal almost Cosymplectic manifold with semi symmetric metric connection together with some of its applications. *Results in this chapter have been published in* **Journal of Dynamical Systems and Geometric Theories**, **17** (1), 2019.

Chapter 8. This chapter is intended to generate some general results for real hypersurfaces in nearly Kaehler 6-Sphere admitting semi-symmetric metric connection. For such real hypersurface, we investigate curvature properties admitting semi-symmetric metric connection and obtained some geometric results. Next, by considering η-Ricci solitons on real hypersurface of nearly Kaehler 6-Sphere deformed by semi-symmetric metric connection, some necessary and sufficient conditions are derived. Later, it is demonstrated that real hypersurfaces of nearly Kaehler 6-Sphere admitting semi-symmetric metric connection with η-Ricci soliton is an η-Einstein real hypersurface. Moreover, it is shown that a real hypersurface is congruent to an open segment of a totally-geodesic hypersphere or a tube over an almost complex curve in 6-Sphere if such a connected real hypersurface of nearly Kaehler 6-Sphere is an η-Ricci soliton with potential vector field ξ. *Results in this chapter have been communicated for publication*.