# M.A./ M.Sc. Mathematics (Self-Financed)

# **SYLLABUS**

# Previous

S. No.	Code	Papers	Internal Assessment	Theory
1.	MT-151	Analysis	25	75
2.	MT-152	Abstract Algebra	25	75
3.	MT-153	Mechanics	25	75
4.	MT-154	Topology	25	75
5.	MT-155	Theory of Differential Equations	25	75
6.	MT-156	Linear Programming and Operations Research	25	75

# Final

S. No.	Code	Papers	Internal	Theory
		•	Assessment	
1.	MT-161	Functional Analysis	25	75
2.	MT-162	Complex Analysis	25	75
6.	MT-163	Linear Algebra	25	75
4.	MT-164	Fluid Dynamics	25	75
5.	MT-165	Differential Geometry	25	75
3.	MT-166	Numerical Analysis	25	75

\* Modification in syllabus as per the approval of BoS.

Department of Mathematics Faculty of Natural Sciences Jamia Millia Islamia New Delhi -110 025

# M.A./ M.Sc. Mathematics (S.F.), Previous

MT-151	Analysis	
Unit 1	Countability of sets. Lebesgue Measure on the real line, Length of intervals, Open and Closed sets on real line. Outer and Inner Lebesgue measure, Lebesgue measurable sets, Properties of measurable sets, Algebra, $\sigma$ -algebra and Borel sets and their measurability, non-measurable sets, Canter's Ternary sets and their properties.	
Unit 2	Measurable functions, Characteristic function, Step function, Continuous function, Set of measure zero, Borel measurable function, The structure of measurable function.	
Unit 3	Riemann integral and its deficiency, Lebesgue integral of bounded function, Comparison of Riemann and Lebesgue integrals, Properties of Lebesgue integral for bounded measurable function, The Lebesgue integral for unbounded functions, Integral of non-negative measurable functions, General Lebesgue integral, Improper integral.	
Unit 4	Pointwise convergence, Convergence almost everywhere, Uniform Convergence almost everywhere, Convergence in measure, F. Reisz's Theorem on Convergence a. e., D.F. Egoroff's Theorem, Lebesgue Bounded Convergence Theorem, Lebesgue Dominated Convergence Theorem, Fatou's Lemma, Monotone Convergence Theorem.	
Unit 5	Dini Derivatives, Differentiation of Monotone Functions, Functions of Bounded Variation, Differentiation of an Integral, Lebesgue sets, Absolute Continuous Functions, Integral of the Derivatives. Lp-space, Normed space, Properties of Lp-space, Lp-space as a normed space, Holder's inequality, Minkowski's inequality and Schwartz's inequality, Convergence in the mean, Cauchy's sequence in the Lp-spaces, Riesz-Fischer Theorem. Bounded Linear functionals of Lp-spaces.	

### **Books Recommended**

Royden, H.L.; Real Analysis (2nd ed.), The Macmillan Co., New York (1968) Jain, P. K., Gupta; Lebesgue measure and Integration, Willey Eastern Ltd., New Age Int. Ltd., New Delhi, (1994).

MT-152	Abstract Algebra	
Unit 1	Groups and its properties, Order of an element of a group, Subgroups, Cyclic groups, Cosets, Normal subgroups and Quotient groups, Lagrange's theorem for finite groups and its applications, Euler's and Fermat's theorem	
Unit 2	Group homomorphism, Isomorphism, Kemel of homomorphism, Fundamental theorem of homomorphism, Permutation groups, Even and odd permutations, Alternating group, Cayley's theorem, Automorphism and Inner automorphism.	
Unit 3	Normalizer and Centre of group, Conjugate class, Class equations and its applications, Direct products, Sylow's theorem and its applications. Finite abelian groups. Dihedral group and Quaternion group.	
Unit 4	Ring, Integral domain, Division ring and Fields, Subring, Ideal and Quotient ring. Types of ideals, Sum and product of ideals	
Unit 5	Homomorphism of rings, Isomorphism, Ring of polynomial and their properties, Euclidean domain, Principal ideal domain, Unique factorization domain, Primitive polynomial, Gauss's lemma, Irreducible polynomial, Eisenstein's criterion for irreducibility.	

### **Books Recommended:**

- 1. L. N. Herstein, Topics in Algebra, Wiley Eastern Ltd., 1975.
- 2. Surjeet Singh and Qazi Zameeruddin, Modern Algebra, Vikas Publications, 2003.
- 3. P. B. Bhattacharya, S.K. Jain, S.R. Nagpal. Basic Abstract Algebra, 1977.
- 4. N. S. Gopalkrishnan, University Algebra, Wiley Eastern, 1986. 5. N Jacobson, Basic Algebra, Hindustan Publishing Co., 1984.

MT-153	Mechanics
Unit 1	Moments of inertia, Kinetic energy, Angular momentum.
Unit 2	Mechanics of a particle and system of particles, kinematics of a rigid body, Euler's angles.
Unit 3	Euler's dynamical equations, two-dimensional motion of a rigid body, Compound pendulum, Constraints.
Unit 4	D'Alembert's principle, Lagrange's equations of motion, Techniques of calculus of variations.
Unit 5	Hamilton's principles, Hamilton's equations of motion, Contact transformation, Lagrange's and Poison brackets, Integral in variances, Hamilton-Jacobi Poisson equations.

### **Books Recommended**

- 1. Principle of Mechanics: Singe and Griffith.
- 2. Lectures in Analytical Mechanics: F. Gantamacher.
- 3. Ele. Treatise on the dynamics of particle and rigid bodies: S. L. Loney.
- 4. A Text Book of Dynamics: F. Choelton.
- 5. An introduction to the Calculus of Variation: C. Fox.
- 6. Calculus of Variations: R. Weinstock.

# MT-154 Topology

- Unit 1 Definition and examples, Open and closed spheres, Open and closed sets, Convergence, Completeness, Cantor's intersection Theorem, Dense sets and separable spaces, Baire's Category Theorem, Continuous mappings, Uniform continuity.
- **Unit 2** Definition and examples, neighbourhood system of a point, Limit points, Closed sets, Closure, Interior and boundary, Bases and sub bases, Continuity, Homeomorphism, Subspaces and product spaces, Local base, First and second countable spaces, Separable spaces, Lindelof's theorem.
- **Unit 3** Definition and examples, Finite intersection property, Heine Borel theorem, Locally compact spaces, Sequential compactness, Bolzano Weierstrass property, Lebesgue covering lemma, Total boundedness.
- Unit 4 Ti (i = 0, 1, 2, 3, 4) spaces, Regular and Completely Regular spaces, Normal and Completely Normal spaces.
- Unit 5 Connected spaces, Components, locally connected spaces, totally connected spaces, totally disconnected spaces, Pathwise connectivity.

# **Books Recommended**

1. G.F. Simmons : Introduction to Topology and Modern Analysis

2. Benjamin T. Sims :Fundamentals of Topology. (Relevant portions only)

- 3. B. Mendelson :Introduction to Topology (Chapters II,III,IV,V) (relevant portions only)
- 4. J. L. Kelley :General Topology.
- 5. W. J. Pervin : Foundations of General Topology.
- 6. J. R. Munkres : Topology

<sup>(</sup>Chapter II 9-13,16-19 IV 21,22,24 V 26,27 VI 31-34)

## **MT-155** Theory of Differential Equations

- **Unit 1** Existence & uniqueness theorem, General theory of homogenous and non-homogenous equations with constant coefficients, Theory of equations with variable coefficients, Method of variation parameter and the formula for particular integral in terms of Wronskian.
- **Unit 2** Series Solution of Second order linear differential equations near ordinary point, Singularity and the solution in the neighbourhood of regular singular point, Euler equation and Frobenious method, Solution of Legendre, Bessel, Hypergeometric, Hermite and Lagurre differential equation.
- **Unit 3** Formulation of Heat conduction equation and its solution by variable separation method, SteadyState condition and the solution of heat conduction problem with non-zero end conditions. Formation of Wave equation and the solution of Wave equation.
- **Unit 4** Linear homogeneous Boundary Value Problems, Eigen values and Eigen functions, Sturm-Liouville Boundary Value Problems, Non-homogeneous Boundary Value Problems, Nonhomogeneous heat conduction problems.
- Unit 5 Green's functions and the solution of Boundary Value Problems in terms of Green's functions, Concept of stability, asymptotic stability and instability of a solution of the autonomous system dx/dt = F(x,y), dy/dt = G(x,y)

### References

- 1. An Introduction to Ordinary Differential Equation: Earl A. Coddington
- 2. Elementary Differential Equations and Boundary Value Problems: Boyce and Diprime.
- 3. A first course in partial differential equations: E. Weinberger

### MT-156 Linear Programming and Operations Research

- **Unit 1** Operations Research (OR) and its Scope. Basic, feasible, optimal, multiple, unbounded and degenerate solution of an LPP. Theory of Simplex method, improved basic feasible solution, condition of optimality. Two-phase simplex method, Big-M method, Duality in linear programming problems, Dual Simplex method.
- **Unit 2** Sensitivity analysis, Integer programming, Branch and Bound method, Gomory's constraint method, Dynamic programming, Bellman's Principle of optimality, Goal programming.
- **Unit 3** Queuing Theory, Transient and Steady states, Queuing model, single server with infinite capacity, multiple servers with infinite capacity, single server with finite capacity, multiple servers with finite capacity.
- **Unit 4** Mathematical formulation of transportation problem, Optimal solution of transportation problem, Assignment problem, Network analysis, Events, Critical Path Method (CPM), Project Evaluation and Review Technique (PERT), Project planning with CPM/PERT.
- **Unit 5** Nonlinear programming problems, One and multi-variable unconstrained optimization, Kuhn-Tucker conditions for constrained optimization, Quadratic programming, Wolfe's Simplex method, Separable programming.

### References

- "Operations Research An Introduction" H. A. Taha.
- "Introduction to Operations Research" Hillier and Lieberman, McGraw Hill.
- "Linear Programming" G. Hadley, Narosa Publishing House.
- "Operations Research" S. D. Sharma, Ram Nath Kedar Nath.
- "Operations Research Theory and Application" J. K. Sharma, Macmillian Pub.
- "Operations Research Problems and Solution" J. K. Sharma, Macmillian Pub.

# M.A./ M.Sc. Mathematics (S.F.) Final MT-161 Functional Analysis

- Unit 1 Definition and examples, Incomplete normed spaces, Completion, Subspaces, Quotient spaces, Schauder basis.
- **Unit 2** Definition and examples, Relation between continuity and boundedness, Null space, Spaces of bounded linear operators, Equivalent norms, Open mapping theorem, Closed graph theorem, Uniform boundedness principle.
- **Unit 3** Definition and examples, Relation between continuity and boundedness, Dual spaces, Duals of  $R^n$ ,  $C^n$ ,  $l^p(n)(1 , <math>C_0$ ,  $l^1$  and  $l^p$  (1 ), Hahn Banach theorem, Embedding and reflexivity, Adjoint operator, Weak and weak<sup>\*</sup> convergence.
- **Unit 4** Definition and examples, Schwartz inequality, Parallelogram equality, Subspaces, Completion, Orthogonality of vectors, Orthogonal complement and projection theorem, Orthogonal sets and Fourier analysis, Complete orthogonal sets.
- **Unit 5** Bounded linear functionals, Riesz-Frechet theorem, Hilbert-adjoint operator, self-adjoint, Normal and Unitary operators, Orthogonal projection operators.

### **Books Recommended**

- 1. P.K. Jain, O.P. Ahuja and Khalil Ahmad: Functional Analysis. New Age International (P), Ltd., New Delhi (1995).
- 2. E. Kreyszig: Introductory Functional Analysis with Applications John-Wiley & Sons, N.Y. -1978

## MT-162 Complex Analysis

- **Unit 1** Representation of Complex Numbers, Analytic Function, Cauchy Riemann Equations, Powers Series, Some Elementary Functions, Harmonic Functions.
- **Unit 2** Properties of Line integral, Zeros of an analytic function, Cauchys Theorem, Cauchy's Integral formula, Cauchy's inequality, Fundamental Theorem of Algebra, Poison's formula, liouville's Theorem, Rouche's Theorem, The argument principle.
- **Unit 3** Residues and Poles, Classification of Isolated singularities, Taylor's and Laurant's Series, Winding Numbers and Cauchy Residue Theorem.
- Unit 4 Application of Residue Theorem in Evaluation of Improper real Integrals and Evaluation of Sum.
- Unit 5 Conformal Mapping Properties, Schwarz Lemma Riemann Mapping Theorem (without proof), Maximum Modulus Theorem, Analytic Continuation.

### **Books Recommended**

1.	Rudin	:	Real and Complex Analysis
2.	J.B. Conway	:	Complex Analysis
3.	Alhfors	:	Complex Analysis
4.	E.C. Titchmarch	:	Complex Analysis
5.	B. Chawdhary	:	Complex Analysis

## MT-163 Linear Algebra

- **Unit 1** Vector spaces, Subspaces and Quotient spaces, Basis and dimension, Direct sum and complements. Independent subspaces. Linear transformation, Rank and Nullity of a linear transformation
- **Unit 2** Algebra of linear transformation, Singular and non-singular linear transformation, Invertible linear transformation, Dual spaces, Principle of duality, Second dual space. Annihilators Unit-III Matrix representation of linear transformation, Linear map associated with a matrix.
- **Unit 3** Change of basis, Similarity and Equivalency of matrices, Isomorphism between linear mappings and matrices, Minimal polynomial, Eigenvalues and Eigenvectors.
- **Unit 4** Inner product spaces, Norm and Normed linear space. Schwarz inequality. Orthogonality and Orthonormality, Gram-Schmidt orthogonalization process, Orthogonal direct sum.
- **Unit 5** Trace and Transpose of a linear transformation, Adjoint of linear operator, Self-adjoint operators, Symmetric and Skew-symmetric operators, Normal operators, Unitary operators, Positive operators. The Spectral theorem.

### **Books Recommended**

- 1. I. N. Herstein. Topics in Algebra, Wiley Easter Ltd., New Delhi, 1975
- 2. K. Hoffman and R. Kunze, Linear Algebra, 2 Edition, Prentice Hall of India, Delhi, 1971.
- 3. P.B. Bhattacharya, S.K. Jain and S.R. Nagpaul, First Course in Linear Algebra, Wiley Eastern, Delhi, 2003
- 4. Surjeet Singh and Q Zameeruddin, Modern Algebra, Vikas Publications, 2003.
- 5. Stephen H. Friedberg, Arnold J. Insel and I. E. Space, Linear Algebra, 4 Edition, PHI Pvt. Ltd., New Delhi, 2004

# MT-164 Fluid Dynamics

- Unit 1 Ideal and Real fluids, Pressure, Density, Viscosity, Description of Fluid motion, o- Lagrangian method, Eulerian method. Steady and unsteady flows, Uniform and nonuniform flows, One dimensional, two dimensional and axisymmetric flows, Line of flows, Stream line Path line, Stream surface, Stream tube, Streak lines, Local and Material delivative, Equation of Continuity.
- **Unit 2** Euler's equation of Motion along a stream line, Equation of motion of an inviscid fluid, conservative field of force, Integral of Eulers equation, Bernoullis equation and its applications, flow from a tank through a small orifice, Cauchys integral, Symmetric forms of the equation of continuity, Impulsive motion of a fluid, Energy equation.
- Unit 3 Dimensional Analysis, Buckinghm's pi theorem, Variable in fluid mechanics, Procedures of dimensional Analysis, Similitude, Important dimension less parameter (Reynold's no., Mech No., Prandtl, Pradtl No. etc.)
- **Unit 4** Navier-Stokes equation, Poseuelles equation for laminar flow in pipe, Stokes low for fall velocity, Darcyslaw, Some simple types of flows (Couette flow and its generalization, Flow between two porous plates)
- **Unit 5** Boundary layer definition and it's characteristics, Leminar boundary layer, Separation and it's control, Similarity solution of boundary layer equation, Boundary layer flow over flat plate, Stagnation point and boundary layer flow near this.

### **Books Recommended**

- 1. Introduction to Fluid Dynamics by R. K. Rathy
- 2. Hydrodynamics by Shanti Sawroop
- 3. Fluid Mechanics by R. J. Garde
- 4. Boundary layer Theory by H. Schlichting

## **MT-165** Differential Geometry

- Unit 1 Coordinate transformation, Covariant, Contravariant and mixed tensors, tensors of higher rank, symmetric and skew symmetric tensor, tensor algebra, Contraction, Inner Product, Quotient Law. Riemannian metric tensor, Christoffel symbols, Transformation Laws of Christoffel symbols, Covariant derivatives of higher rank tensor, Riemannian curvature tensor.
- **Unit 2** Differentiable curves and their parametric and implicit representations, Tangent vector, Principal normal, Binormal, curvature and torsion, Serret-Frenet formulas, Fundamental theorem for space curve. Vector fields, Covariant differentiations, Connexion forms and structural equations in E<sup>3</sup>.
- **Unit 3** Curvilinear Co-Ordinates on a Surface, First fundamental forms, Geodesic on surface, Geodesic co-ordinates.
- **Unit 4** Second fundamental forms, Tensor derivative, Gauss-Weingarten formulae, Integrability condition, Gauss & Mainardi Codazzi equations, Meusrier theorem, Geodesic curvature.
- **Unit 5** Line of curvature, Asymptotic lines, Gauss and mean curvature, Minimal surfaces, Third fundamental forms.

### **Books Recommended**

- 1. Introduction to Differential Geometry: Abraham Goetz; Addison Wesley Pub. Company.
- 2. Differential Geometry: Nirmala Prakash; McGraw-Hill
- 3. Elementary Differential Geometry: B.O. Neill; Academic Press.
- 4. A course in tensors with Application to Riemannian Geometry: R.S. Mishra
- 5. An introduction to Differential Geometry: T.J. Willmore
- 6. Introduction to Riemannian Geometry and Tensor Calculus: Weitherburn

## MT-166 Numerical Analysis

- **Unit 1** Solution of transcendental and polynomial equations by Newton-Raphson method for simple and multiple roots, Rate of convergence, Solution of system of non-linear equations by Iteration and Newton-Raphson method, Method for Complex roots.
- **Unit 2** Lagrange's form of interpolating polynomial, Existence and uniqueness of interpolating polynomial, Hermite, Piecewise and Cubic spline interpolation. Approximation: Weighted least squares approximation, Method of least squares for continuous functions, Gram-Schmidt orthogonalization process and approximation of functions using Chebyshev polynomials.
- Unit 3 Numerical integration: Guass Quadrature methods and error estimation. Double integrations, Numerical solution of Initial Value Problems: Runge-Kutta method of order four for system of equations and for second and higher order differential equations. Boundary Value problems by Finite difference methods and shooting method. Convergence and stability of finite difference scheme.
- **Unit 4** Numerical solution of partial differential equations: Parabolic equations- finite difference approximation to partial derivatives, explicit method and Crank-Nicolson method with stability analysis. Elliptic equations- Standard five-point formula, Jacobi's iteration method and Leibmann's method. Hyperbolic equations: Explicit finite difference method.
- **Unit 5** Finite element methods for ordinary differential equations: variational methods. method of weighted residuals, finite element analysis of one and two-dimensional problems.

### **Books:**

- 1. S.D. Conte & Carl De Boor, Elementary Numerical Analysis, McGraw Hill, NY
- 2. M. K. Jain, S.R.K Iyengar and R. K. Jain, Numerical Methods for Scientific and Engineering Computations, New Delhi.
- 3. G.D. Smith, Numerical Solutions of Partial Differential Equations, Clarendon Press Oxford
- 4. J. N. Reddy, An Introduction to Finite Element Method, McGraw Hill, 2005. Estimation
- 5. Naseem Ahmad, Fundamentals Numerical Analysis with Error Estimation, Anamaya Publishers
- 6. Gerald & Wheatlay: Applied Numerical Analysis, Pearson