M.A./ M.Sc. Mathematics (Self-Financed)

Course Structure and Syllabus

(w.e.f. 2024-25)

3rd Semester

S. No.	Code	Papers	Credits
1.	MAS-301	Advanced Ring Theory	4
2.	MAS-302	Functional Analysis	4
3.	MAS-303	Differential Geometry	4
4.	MAS-304	Mathematical Statistics and Random Processes	4
5.	MAS-305	One paper from elective 1	4

Elective 1: *Select one paper*,
MAS-305E1: Sobolev Spaces and Applications
MAS-305E2: Algebraic Number Theory
MAS-305E3: Data Science using Python

MAS-301 Advanced Ring Theory L/T/P: 4/0/0

Unit-1: Definition, types and fundamental properties of rings (Review), Boolean rings, Nilpotent and idempotent elements, Characteristic of rings, Subrings and ideals, Sum and direct sum of ideals, Homomorphisms and embedding of rings, Minimal and maximal ideals, Factor rings.

Unit-2: Principal ideal and ideals generated by a subset, Nilpotent ideals and nil Ideals, Complete matrix ring and subdirect sum complete matrix ring, Ideals in complete matrix ring, Subdirect sum of rings and its characterizations, Zorn's Lemma, Subdirectly irreducible rings.

Unit-3: Prime ideals and m-systems, Different equivalent formulation of prime ideals, Semi-prime ideals and n-systems, Equivalent formulation of semi-prime ideals, Necessary and sufficient conditions for an ideal to be a prime ideal, Prime radical of a ring.

Unit-4: Prime rings, Jacobson radical prime rings and its characterization in terms of prime ideals, Primeness of complete matrix rings, Descending chain condition for ideals and the prime radical, Jacobson radical, Relationship between Jacobson radical and prime radical of a ring, Primitive rings, Jacobson radical of primitive rings.

Books Recommended

1. N. H. McCoy: *The Theory of Rings*, Macmillan, New York, 1964.
2. I. N. Herstein, *Topics in Algebra*, 3rd ed., Wiley, New York, 1996.
3. D. S. Dummit and R.M. Foote, *Abstract Algebra*, John Wiley, New York, 2004.
4. J. A. Gallian, Contemporary *Abstract Algebra*, Narosa Publ. House, New Delhi, 1998.
5. S. Singh and Q. Zameeruddin: *Modern Algebra*, Vikas Publ. House, New Delhi, 2002.
6. I. S. Luthar and I. B. S. Passi, *Algebra, Vol. 2: Rings*, Narosa Pub. House, New Delhi, 1999.
7. F. W. Anderson and K. R. Fuller: *Rings and Categories of Modules*, Springer-Verlag, New York, 1992.

MAS-302 Functional Analysis L/T/P: 4/0/0

Unit-1: Definition and example of norm linear space, Completeness of norm linear spaces, Subspaces, Finite dimensional norm linear spaces and their properties, Reisz's Lemma, Equivalent norm, Completion.

Unit-2: Definition and examples of bounded linear operators, Relation between continuity and boundedness, Null space, Spaces of bounded linear operators, Open mapping theorem with applications, Closed graph theorem and its consequences, Uniform boundedness principle.

Unit-3: Definition and examples of bounded linear functional, Dual spaces and their properties, Computation of duals of slandered norm linear spaces, Hahn Banach theorem and its consequences, Embedding and reflexivity, Weak and weak* convergence.

Unit-4: Banach contraction theorem, Applications of Banach contraction theorem to linear equations, Applications of Banach contraction theorem to initial value problems,

Picard's existence and uniqueness, Applications of Banach contraction theorem to Fredholm and Volterra Integral equations.

Books Recommended

1. E. Kreyszig: *Introductory Functional Analysis with Applications* John-Wiley & Sons, N.Y. -1978
2. P.K. Jain, O.P. Ahuja and Khalil Ahmad: *Functional Analysis*. New Age International (P), Ltd., New Delhi (1995).

MAS-303 Differential Geometry L/T/P: 4/0/0

Unit-1: Coordinate transformations, covariant, contravariant and mixed tensors, tensor of high rank, symmetric and skew symmetric tensors, tensor algebra, contraction, inner product, Quotient law, Riemannian metric tensor, Christoffel symbols, transformation law of Christoffel symbols, covariant derivative of high rank tensors. Riemannian curvature tensor

Unit-2: Parametrization of curve, unit tangent, normal and binormal vectors, tangent line, normal line and binormal line, normal plane, rectifying plane, osculating plane to the curve, curvature and torsion, Frenet serret equations, fundamental theorem for space curve, vector fields, covariant differentiations, connection forms and structural equations in E^3.

Unit-3: Regular parametrization of the surface, curvilinear coordinate, coordinate patches, tangent plane and normal line to the surface, First and second fundamental forms and their applications, principal curvature, normal curvature, Gaussian curvature and mean curvature to the surface, line of curvature, Rodrigues formula and minimal surfaces.

Unit-4: Geodesic and Geodesic differential equations, Geodesics curvature, Shape operator and its properties, fundamental equations of surface theory, Gauss and Weingarten equations, Mainardi Codazzi equation.

Books Recommended

1. Martin M. Lipschutz, *Theory and Problems of Differential Geometry*, McGraw Hill, 1969.
2. A. Goetz, *Introduction to Differential Geometry*, Addison Wesley Pub. Company
3. Barrett O. Neill, *Elementary Differential Geometry* Academic Press Elsevier, 2006
4. R. S. Mishra, *A course in tensors with Application to Riemannian Geometry*, Pothishala Private Ltd Allahabad,1965
5. T. J. Wilmore, *An Introduction to Differential Geometry*, Dover Publications Inc. Mineola, NY, 1969
6. C. E. Weatherburn, *Introduction to Riemannian Geometry and Tensor Calculus*, Cambridge press, 2008

MAS-304 Mathematical Statistics and Random Processes L/T/P: 4/0/0

Unit-1: Quantitative and qualitative data, variables and attributes data. Scales of measurement: nominal, ordinal, interval, and ratio. Measures of Central Tendency, Measures of Dispersions, Skewness and Kurtosis. Bi-variate data, Correlation (simple, partial and multiple), rank correlation, and Regression Analysis. Principle of least squares and fitting of polynomials and exponential curves.

Unit-2: Review of one and two-dimensional random variables: moments, moments generating functions and characteristics functions. Random vectors and its distributions. Chebyshev inequality and convergence in probability, Markow and Jansen inequalities. Transformations of random variables and Central limit theorems.

Unit-3: The Analysis of Variance (ANOVA): Single-Factor ANOVA, Multiple Comparisons in ANOVA, Two-Factor ANOVA with $K_{ij} = 1$ and $K\_ij > 1$ Three-Factor ANOVA and 2^p Factorial Experiments. Nonlinear and Multiple Regressions: Assessing Model Adequacy, Regression with Transformed Variables, Polynomial Regression, Multiple Regression Analysis.

Unit-4: Random processes and their classification, Analytical representation of a random process, Stationary processes, Random telegraph signal, Autocorrelation and cross-correlation functions, Ergodicity of random process. Bernoulli process, Sine wave process, Markov process, Markov chain, Binomial. Poisson and normal (Gaussian) processes, Birth, and death process. Spectral analysis of random processes.

Books Recommended

1. S. C. Gupta and V.K. Kapoor, "*Fundamentals of Mathematical Statistics*", Sultan Chand & Sons, New Delhi.
2. N. H. Timm, "*Applied Multivariate Analysis*", Springer, New York.
3. Jay Devore, "*Probability and Statistics for Engineering and the Sciences*", Cengage Learning, USA.
4. T. Veerarajan, "*Probability, Statistics and Random Processes*", Tata McGraw Hill, New Delhi.

Electives

MAS-305E1 Sobolev Spaces and Applications L/T/P: 4/0/0

Unit-1: Distribution Theory: Test function spaces, distributions, calculus on distributions and basic examples, Localization, supports of distributions and local structure of distributions. Convolutions, convolution of distributions, fundamental solutions, Fourier transform, and tempered distributions.

Unit-2: Sobolev Spaces: Definition and basic properties. Approximation by smooth functions, Poincare inequality, imbedding theorems, extension theorems. Sobolev inequalities, compactness theorems, trace theory.

Unit-3: Applications to Partial Differential Equations: Variational problems in Hilbert spaces and Lax-Milgram lemma. Second-order elliptic equations, weak solutions, existence of weak solutions, Examples of weak formulations of elliptic boundary value problems.

Unit-4: Regularity theory, maximum principles and eigen-value problems, Introduction to the Finite Element Method (FEM) and Spectral Element Method (SEM).

Books Recommended:

1. S. Kesavan. *Topics in Functional Analysis and Applications*. Wiley Eastern Limited, (1989). ISBN: 978-0470210505
2. L. C. Evans. *Partial Differential Equations (2nd Edition),* AMS, (2010). ISBN: 978-0821849743
3. R. Adams, J. Fournier: *Sobolev spaces*, Academic Press, Elsevier, (2003). ISBN: 9780120441433
4. G. Leoni. *A First Course in Sobolev Spaces*. AMS, (2009). ISBN: 978-1470411695
5. D. Gilbarg and N. Trudinger. *Elliptic Partial Differential Equations of Second Order* (2nd Edition), Springer, (1983).

MAS-305E2 Algebraic Number Theory L/T/P: 4/0/0

Unit-1: Algebraic numbers, transcendental numbers, minimal polynomials, conjugates. Number fields, primitive elements, real and complex embeddings, norm, trace and discriminant.

Unit-2: Algebraic integers, Ring of integers in a number field, Integral basis. Dedekind domain, Ideals in a Dedekind domain and their factorization into prime ideals, Norm of an ideal, Norm of product of ideals.

Unit-3: Fractional ideal, Ideal class group, Class number, Finiteness of the ideal class group, Minkowski bound, Algorithm to determine the ideal class group and examples.

Unit-4: Theory of valuations, Dirichlet unit theorem, Fundamental units, Units in quadratic fields, Pell's equation, Cyclotomic fields.

Books Recommended

1. Richard A. Mollin, *Algebraic Number Theory*, CRC Press, 1999.
2. Neukirch, *Algebraic Number Theory*, Springer, 1999.
3. Stewart and Tall, *Algebraic Number Theory and Fermat's Last Theorem*, A K Peters, 2002.
4. Marcus, D. A., *Number Fields*, Springer, 1977.
5. Alan Baker, A Concise, *Introduction to Algebraic Number Theory*, Cambridge University Press,1994.
6. Janusz G.J., *Algebraic Number Fields*, GSM, vol-7, AMS, 1996.

MAS-305E3 Data Science using Python L/T/P: 3/0/2

Unit-1: Foundation of Data Science - Definition, scope, applications of data science, data science life cycle, data science frameworks. Data Collection: data acquisition from various sources (databases, APIs, web scraping), data pre-processing. Introduction to Python Programming: installation and setup, keywords, variables, identifiers, functions, string handling.

Unit-2: Data exploration using Python - Overview of list, tuples and dictionaries, exploring arrays with NumPy, *n*-dimensional arrays operations and manipulations, creating arrays from ranges, slicing and indexing. Exploratory data analysis using Pandas: introduction to data frames, summary statistics, exploratory techniques for understanding data distributions, merging and joining data frames, reshaping datasets.

Unit-3: Data plotting and Feature engineering - Two-dimensional plotting in Matplotlib, line plot, simple scatter plot, histograms, three-dimensional plotting. Feature

identification: handling categorical variables, encoding schemes, feature scaling, normalization and transformation, feature selection and reduction using Principal Component Analysis, correlation analysis.

Unit-4: General approaches for classification - Overview of classification algorithms using Python's scikit-learn library. Natural Language Processing (NLP): stemming, lemmatization, parts of speech tagging, chunking, syntax parsing, text tokenization, BoW, TF-IDF, word cloud, word embeddings (Word2Vec, GloVe). Overview of public NLP toolkits. Introduction to sentiment analysis.

Books Recommended

1. A. N. Kamthane, Programming and Problem Solving with Python, 2nd Edition, McGraw Hill Education, 2020.
2. Sameer Madhavan, Mastering Python for Data Science, Packt Publishing Limited, 2015.
3.Wes McKinney, Python for Data Analysis, 2nd Edition, O'Reilly Media, 2017.
4. Dipanjan Sarkar, Text Analytics with Python: A Practical Real-World Approach to Gaining Actionable Insights from Your Data, Apress, 2016.
5. Jiawei Han, Micheline Kambers and Jian Pei, Data Mining Concepts and Techniques, 3rd Edition, Morgan Kaufman Publications, 2012.
6.Aurélien Géron, Hands-On Machine Learning with Scikit-Learn, Keras, and TensorFlow, O'Reilly Media, Inc., 2nd Edition, 2019.