M.A./ M.Sc. Mathematics (Self-Financed)

Course Structure and Syllabus

(w.e.f. 2024-25)

4th Semester

S. No.	Code	Papers	Credits
1.	MAS-401	Differentiable Manifolds	4
2.	MAS-402	Advanced Functional Analysis	4
3.	MAS-403	Field Theory and Galois Theory	4
4.	MAS-404	Graph Theory	4
5.	MAS-405	One paper from elective 2	4

Elective 2: *Select one paper*

MAS-405E1: Wavelet Analysis

MAS-405E2: Fluid Dynamics

MAS-405E3: Machine Learning

MAS-401 Differentiable Manifolds L/T/P: 4/0/0

Unit-1: Differentiable manifolds, compatible atlas, differentiable atlas, product manifolds, smooth functions on a manifolds, tangent vector and tangent space, vector fields, Lie Bracket and properties.

Unit-2: Dual space, convectors, tensors and forms, pullback of p-forms, tensor product, law of transformations, algebra of tensors, p-forms, wedge products, Grassman algebra, exterior differential operator,

Unit-3: Connection on a manifold, parallelism and geodesics, torsion of a connection, properties of curvature and torsion tensors, Bianchi' Identities.

Unit-4: Riemannian manifolds, Riemannian metric, Fundamental Theorem of Riemannian Geometry, Riemannian curvature tensor, Differentiable distributions, and Integrability conditions.

Books Recommended

1. Y. Matsushima, *Differentiable Manifolds*, 1972
2. K. Yano and M. Kon, *Structures on Manifolds*, World Scientific,1985
3. S. S. Chern, *Differentiable Manifolds (Lecture Notes)*, Department of Mathematics, University of Chicago, 1953
4. S.T. Hu, *Differentiable Manifolds*, Holt, Rinehart and Winston, First Edition, 1969
5. J.M. Lee, *Manifolds and Differential Geometry*, Texas Tech University, AMS, 2009
6. C. E. Weatherburn, *Introduction to Riemannian Geometry and Tensor Calculus*, Cambridge press, 2008

MAS-402 Advanced Functional Analysis L/T/P: 4/0/0

Unit-1: Hilbert spaces, Parallelogram law, polarization identity, Schwartz inequality, Orthogonal and orthonormal sets, total orthonormal sets, projection theorem, Bessel's inequality, generalized form, reflexive spaces, separable spaces

Unit-2: Hilbert adjoint operator and its existence; Relation between adjoin and Hilbert adjoint operators, Riesz representation theorem; Self-adjoint operators; convergence of sequence of self-adjoint operators, normal operators, unitary operators.

Unit-3: Spectral theory of operators: Eigen values and eigen vectors, resolvent operators, spectrum, spectral properties of bounded linear operators: self-adjoint operators, properties of resolvent and spectrum, spectral radius, compact linear operators

Unit-4: Unbounded linear operators; Hellinger-Toeplitz theorem; Symmetric operators; Matrix transformations.

Books Recommended

1. E. Kreyszig: *Introductory Functional Analysis with Applications*, John Wiley and Sons, New York, 1989.
2. G Bachman and L. Narici: *Functional Analysis*, Academic Press, New York, 1966.
3. M. Thamban Nair: *Functional Analysis, A First Course*, PHI Learning Pvt Ltd, New Delhi, 2010.
4. P.K. Jain, O.P. Ahuja and K. Ahmad: *Functional Analysis*, New Age International (P) Ltd., New Delhi, 1995.

MAS-403 Field Theory and Galois Theory L/T/P: 4/0/0

Unit-1: Definition of field extensions, Algebraic elements, Algebraic extensions, Finite extensions, Degree of an algebraic element, Minimal polynomial, Degree of a field extension, Extension of a field obtained by adjoining one algebraic element, Splitting field for a set of polynomials, Composition of two sub extensions of an extension, Existence of algebraic closure.

Unit-2: Separable elements, Separable extensions in characteristic 0 all extensions are separable, Separable extensions form a distinguished class, Existence of separable closure, Normal extensions, Finite fields: existence and uniqueness.

Unit-3: Galois extensions, Galois groups, Subgroups, Fixed fields, Galois correspondence, Fundamental theorem of Galois theory, Galois theory – Sylow theory proof that the field of complex numbers is algebraically closed.

Unit-4: Straight edge and compass constructions, Impossibility of trisection of angle $\pi/3$, Solvability by radicals in terms of Galois group, Insolvability of a general quintic.

Books Recommended

1. Artin, E.: *Galois Theory*, University of Notre Dame Press, 2nd Edition, 1944
2. Serge Lang: *Algebra*, Revised 3rd Edition.
3. Joseph Rotman: *Galois Theory*, 2nd Edition, 1998.
4. David S. Dummit, Richard M. Foote: *Abstract Algebra*, 2nd Edition, 1999

MAS-404 Graph Theory L/T/P: 4/0/0

Unit-1: Graphs and Digraphs-Definition, Examples and its different types; Degree of Vertex and related properties; Subgraphs; Walks, Trails, Paths and circuits; Connected Graphs; Disconnected Graphs; Components; Partite Graphs; Operations on Graphs; Graphs Isomorphism

Unit-2: Eulerian Tours and Circuit; Eulerian Graphs and their Applications; Hamiltonian Graphs and their applications; Line Graphs; Factorizations; Cut points, Bridges and Blocks; Block graphs; Trees; Binary Trees; Spanning trees; Shortest Paths Problems.

Unit-3: Planner graphs - Definitions, Examples, and Properties; Euler's Theorem; Kuratowski's Theorem; Graph Coloring - The Chromatic Number; Five Color Theorem; Four Calor Conjecture.

Unit-4: Representation of graphs - Adjacency Matrix; Incidence Matrix; Circuit Matrix Fundamental Circuit Matrix and Rank; Application to a switching Network; Cut set Matrix.

Books Recommended

1. N. Deo: *Graph Theory with applications to Engineering and Computer Science*; PHI.
2. Harary: *Graph Theory*, Narosa Pub House.
3. D. B. West, *Introduction to Graph Theory* (2 ed.) Narosa Pub House, New Delhi.
4. Gary Chartranel and Ring Zhang: *Introduction to Graph Theory*; Tata McGraw Hill Ed.
5. K. R. Parthasarthy: *Basic Graph Theory*, Tata Mac Graw Hill (1994).
6. R. J. Wilson: *Introduction to Graph Theory* (4 ed.) Pearson Education Singapore (2003).

Electives

MAS-405E1 Wavelet Analysis L/T/P: 4/0/0

Unit-1: Fourier transform in L1(R), Properties of Fourier transforms, Fourier transform in L2(R), Parseval Identities, Change of root, Inversion formula, Plancherel Theorem, Duality Theorem, Poisson summation formula, Sampling theorem, Heisenberg's uncertainty principle, Heisenberg's inequality, Discrete Fourier transform, Fast Fourier transform.

Unit-2: Wavelet Transform: Gabor transform, Parseval formula, Inversion formula, Continuous wavelet transform, Mexican hat wavelet, Properties of wavelet transforms, Discrete wavelet transform.

Unit-3: Multiresolution Analysis and Construction of Wavelets: Multiresolution Analysis, Mother wavelet, Haar wavelet, Shannon wavelet, Meyer wavelet, Franklin wavelet, Orthonormal spline wavelets, compactly supported wavelets.

Unit-4: Wavelets and Applications: Biorthogonal wavelets, Wavelets in several variables, Wavelet packets, Multiwavelets, Wavelet frames, Applications in Neural Networks, Turbulance and Medicine

Books Recommended

1. D. F. Walnut, *An Introduction to Wavelet Analysis*, Birkhauser, Boston, 2002.
2. C. K. Chui, *Wavelets: A Tutorial in Theory and Applications*, Academic Press, 1992.
3. Khalil Ahmad and F. A. Shah, *Introduction to Wavelets with Applications*, World Education Publishers, 2012.

MAS-405E2 Fluid Dynamics L/T/P: 4/0/0

Unit-1: Classification of fluids, Description of Fluid motion- Lagrangian method, Eulerian method. Irrotational flow, Streamlines, Path lines, Streak line of the particles, Stream tube, Stream surface. Steady and unsteady flows, Uniform and non-uniform flows, Local and Material derivatives, Equation of Continuity.

Unit-2: Euler's equation of Motion along a stream line, Equation of motion of an inviscid fluid, conservative field of force, Integral of Euler's equation under different conditions, Bernoulli's equation and its applications, flow from a tank through a small orifice, Energy equation, Impulsive motion of a fluid.

Unit-3: Navier-Stokes equation, Poiseuelle's equation for laminar flow in pipe, Stoke's law for terminal fall velocity, Darcy's law, some simple types of flows (Plane Couette flow, Plane Poiseuille flow, Hagen-Poiseuille and Blasius flow).

Unit-4: Dimensional Analysis, Rayleigh's technique Buckingham's pi theorem, Significance of Reynold's number, Variable in fluid mechanics, Similitude, Froude number, Euler number, Mach number, Prandtl number. General outline of boundary layer flow, Boundary layer thickness, Displacement thickness, Energy thickness.

Books Recommended

1. D. E Rutherford: *Fluid Dynamics*, Oliver and Boyd,
2. G. Emanuel: *Analytical Fluid Dynamics*, CRC Press; 3rd edition, 2015
3. F. Chorlton: *Textbook of Fluid Dynamics*, CBS, 1985
4. R. J. Garde: *Fluid Mechanics*, New Age Publisher, 2011
5. M. D. Raisinghania: *Fluid Dynamics*, S Chand & Company; 5th edition, 2003

MAS-405E3 Machine Learning L/T/P: 3/0/2

Unit-1: Foundations of machine learning - Definition and scope of machine learning, machine learning paradigms, machine learning applications, classification and regression analysis, model building and evaluation, cross-validation, evaluation metrics, overview of public machine learning toolkits.

Unit-2: Supervised learning- Linear models: Naïve Bayes, Linear regression. Regularization techniques: Lasso, Ridge, Logistic regression. Non-linear Models: Decision trees and ensemble methods, Random Forests, Gradient Boosting, Support Vector Machines, K-Nearest Neighbours (KNN), hyperparameter tuning.

Unit-3: Unsupervised learning - Requirement for cluster analysis, overview of basic clustering methods: Partitioning methods, Hierarchical clustering, Density-based methods, Grid-based methods. Association rule learning: Apriori algorithm for mining association rules, market basket analysis and its applications.

Unit-4: Neural Networks and Deep Learning - Introduction to neural networks, multilayer perceptron, feedforward, backpropagation, activation function, optimization methods, Convolutional Neural Networks (CNNs), Recurrent Neural Networks (RNNs), Long Short-Term Memory (LSTM), Bidirectional LSTM, overview of large language models, contextual embeddings and transfer learning using pre-trained language models.

Books Recommended

1. Mehryar Mohri, Afshin Rostamizadeh, Ameet Talwalkar, *Foundations of Machine Learning*, MIT Press, 2018.
2. Tom M. Mitchell, *Machine Learning*, McGraw Hill, 3rd Edition, 2017.
3. Charu C. Aggarwal, *Data Classification: Algorithms and Applications*, CRC Press, 2014.
4. Stephen Marsland, *Machine Learning - An Algorithmic Perspective*, 2nd Edition, CRC Press, 2015.
5. Kevin P. Murphy, *Machine Learning: A Probabilistic Perspective*, The MIT Press, 2012.
6. Ian Goodfellow, Yoshoua Bengio, and Aaron Courville, *Deep Learning*, MIT Press, 2016.